Highly Precise Gravitational Waveform

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Matched Filtering

$$s(t) = h(t) + n(t)$$

$s(t)$: Strain amplitude of detector

$h(t)$: Gravitational Wave Signal

$n(t)$: Noise

$$|h(t)| \ll |n(t)|$$

1. Motivation

Matched Filtering

$$\frac{1}{T}\int_0^T dt\, s(t)h(t) = \frac{1}{T}\int_0^T dt\, h(t)^2 + \frac{1}{T}\int_0^T dt\, n(t)h(t)$$

As $T \to \infty$, $$= h_o^2 + \frac{h_o n_o}{T^{\frac{1}{2}}}$$

Filtered Out

1. Motivation

Matched Filtering

$$\frac{1}{T}\int_0^T dt\, s(t)K(t) = \frac{1}{T}\int_0^T dt\, h(t)K(t) + \frac{1}{T}\int_0^T dt\, n(t)K(t)$$

$$\frac{S}{N} \sim \frac{\frac{1}{T}\int_0^T dt\, h(t)K(t)}{\frac{1}{T}\int_0^T dt\, n(t)K(t)}$$

Signal to Noise ratio

1. Motivation

Bayesian Inference

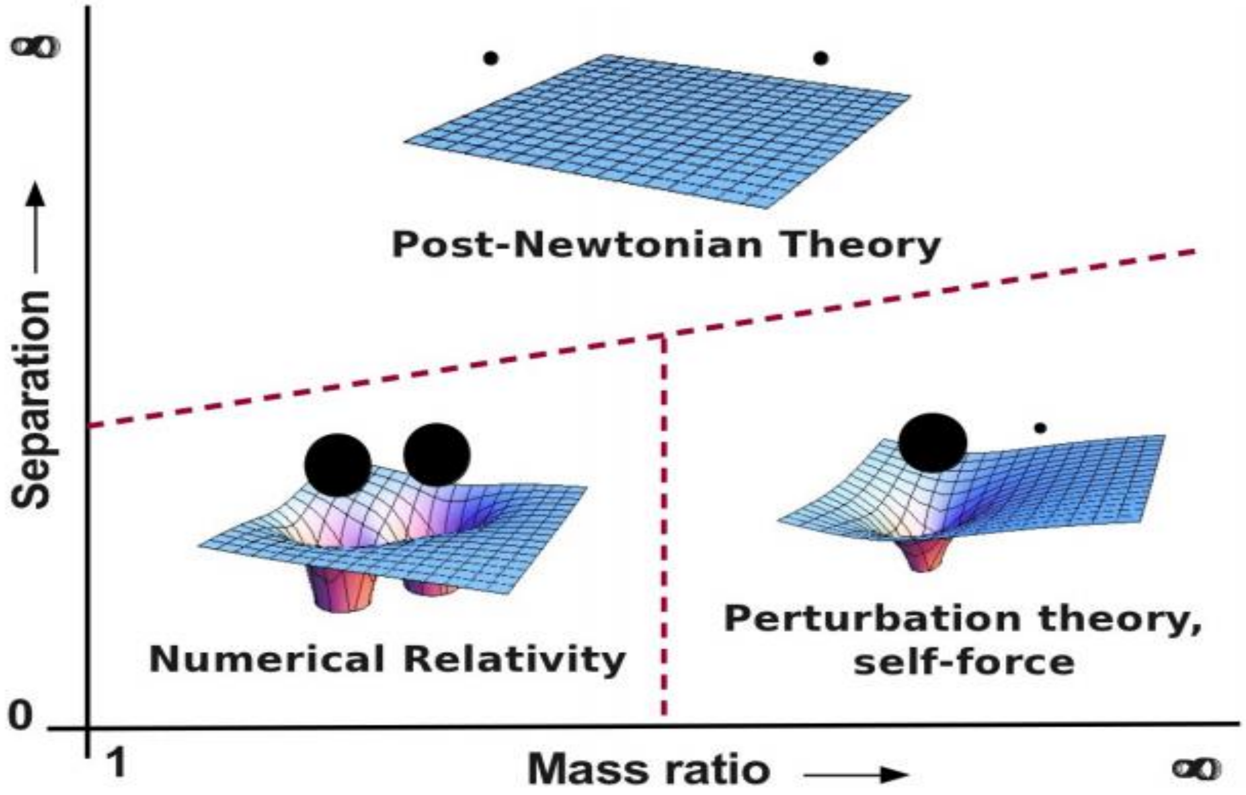
$$p(H_i|DI) = \frac{p(H_i|I)p(D|H_iI)}{p(D|I)} \tag{12}$$

where each term is given a special name:

- $p(H_i|DI)$ is the posterior probability for H_i;
- $p(H_i|I)$ is the prior probability for H_i;
- $p(D|H_iI)$ is the likelihood function for the data D given H_i. Sometimes this quantity is also referred to as *sampling probability* for D;
- $p(D|I) = \sum_i p(H_i|I)p(D|H_iI)$ is (for the moment) a normalisation factor to ensure that $\sum_i p(H_i|DI) = 1$.

2. Methods for Gravitational Waveforms

Binary Parameter space



[Leor Barack]

3. Post-Newtonian theory

An Expansion of Gravitational Field and Matter field in the powers of $\frac{1}{c}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h^0_{\mu\nu} + \frac{1}{c} h^1_{\mu\nu} + \frac{1}{c^2} h^2_{\mu\nu} + \frac{1}{c^3} h^3_{\mu\nu} + \cdots$$

$$T_{\mu\nu} = T^0_{\mu\nu} + \frac{1}{c} T^1_{\mu\nu} + \frac{1}{c^2} T^2_{\mu\nu} + \frac{1}{c^3} T^3_{\mu\nu} + \cdots$$

3. Post-Newtonian theory

Meaning of the Perturbation

Slowly Moving Matter

Self gravitating $\longrightarrow$ Weak Gravity $\longrightarrow$ Large separation

Small Retardation $F\left(t-\frac{r}{c}\right)=F(t)-\frac{r}{c}F'(t)+\frac{1}{2}\left(\frac{r}{c}\right)^2F''(t)+\cdots.$

3. Blackhole Encounter and Burst Waveform

Generalized quasi-Keplerian parametrization for compact binaries in hyperbolic orbits

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We derive third post-Newtonian (3PN) accurate Keplerian-type parametric solution to describe PN-accurate dynamics of non-spinning compact binaries in hyperbolic orbits. The orbital elements and functions of the parametric solution are obtained in terms of the conserved 3PN accurate conserved orbital energy and angular momentum in both Arnowitt-Deser-Misner type and modified harmonic coordinates. Elegant checks are provided that include a modified analytic continuation prescription to obtain our independent hyperbolic parametric solution from its eccentric version. A prescription to model gravitational wave polarization states for hyperbolic compact binaries experiencing 3.5PN-accurate orbital motion is presented that employs our 3PN-accurate parametric solution.

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I. INTRODUCTION

Interesting astrophysical scenarios involving strong gravitational fields usually require accurate and efficient ways of describing compact binary orbital dynamics. These scenarios include gravitational wave (GW) events, observed by the advanced LIGO -Virgo interferometers [1], labeled GW150914, GW151226, GW170104, GW170814 and GW170817 [2–6]. The first four are associated with the coalescence of black hole (BH) binaries while GW170817 involved a merging neutron star binary. The other strong field scenarios involving compact binaries include radio observations of relativistic binary pul-

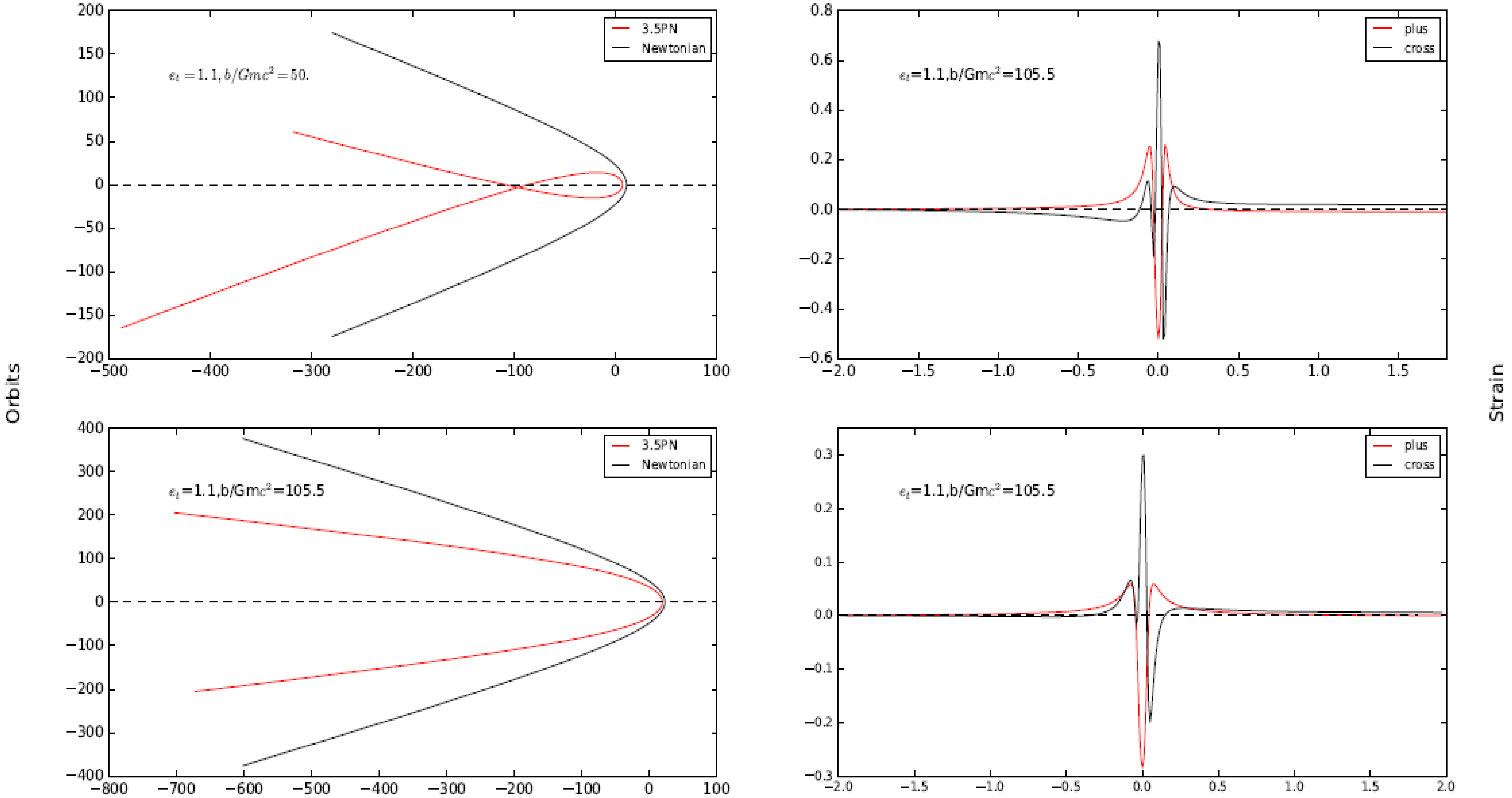
centricities so that the parametrization looks 'Keplerian' even at 1PN order. These computations were extended to 2PN and 3PN orders by Schäfer and his collaborators which led to the generalized quasi-Keplerian parametric solution for compact binaries in precessing eccentric orbits [17–19]. This solution plays an important role in the on-going efforts to model GWs associated with merging BH binaries in eccentric orbits [20, 21]. This is due to the use of certain GW phasing formalism, developed in Refs. [22, 23], for describing the inspiral part of eccentric binary coalescence. This formalism employs Keplerian-type parametric solution to model orbital and periastron precession timescales variations present in the two GW

3. Blackhole Encounter and Burst Waveform

Analytic Solution for 3PN accurate conservative part of hyperbolic encounter

Numerical Inclusion of 2.5PN and 3.5PN accurate dissipative part of hyperbolic encounter

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{0}_{\mu\nu} + \frac{1}{c} h^{1}_{\mu\nu} + \frac{1}{c^2} h^{2}_{\mu\nu} + \frac{1}{c^3} h^{3}_{\mu\nu} + \frac{1}{c^4} h^{4}_{\mu\nu} + \frac{1}{c^5} h^{5}_{\mu\nu} + \frac{1}{c^6} h^{6}_{\mu\nu} + \frac{1}{c^7} h^{7}_{\mu\nu}.$$



Orbits
Strain
3.5PN
Newtonian
$\epsilon_t = 1.1, b/Gmc^2 = 50.$
ϵ_t=1.1,b/Gmc^2=105.5
plus
cross
ϵ_t=1.1,b/Gmc^2=105.5
3.5PN
Newtonian
ϵ_t=1.1,b/Gmc^2=105.5
plus
cross
ϵ_t=1.1,b/Gmc^2=105.5

4. Blackhole Capture

The third post-Newtonian energy and angular momentum flux for compact binaries in hyperbolic orbits

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We compute the third post-Newtonian (3PN) energy and angular momentum flux for compact binaries in hyperbolic orbits.

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I. INTRODUCTION

II. THE HYPERBOLIC MOTION

A. Quasi-Keplerian parametrization

Here we give the 3PN accurate quasi-Keplerian parametrization of the hyperbolic motion.

B. Fourier decomposition of the multipole moments

The quasi-Keplerian parametrization of the hyperbolic orbits allows us to rewrite the multipole moments of the binary system in the Fourier domain. There are two important differences compared to the eccentric case. First of all there is no periodic motion, so we cannot use a discrete 2π periodic Fourier series. We thus need to cal-

quadrupole moment and is given at Newtonian order by

$$I_{ij} = \mu x_{\langle i} x_{j\rangle} \,. \tag{3}$$

We decompose the quadrupole moment into its Fourier components

$$I_{ij}(t) = \int_{-\infty}^{\infty} \tilde{I}_{ij}(p) e^{ipl} dp \,, \tag{4}$$

with the Fourier transform given as in Eq. (1). This allows us to write the energy flux as

$$\mathcal{F}^{(N)} = \frac{1}{5} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dp\, dq\, (ipn)^3 (iqn)^3 \times \tilde{I}_{ij}(p) \tilde{I}_{ij}(q) e^{i(p+q)l} \,. \tag{5}$$

We now integrate this over the whole orbit, using the fact that $\langle e^{ipl} \rangle = 2\pi\delta(p)$. Note also that $\tilde{I}_{ij}(-p) = \tilde{I}_{ij}(p)^*$. We thus find

4. Blackhole Capture

$$\mathcal{F}_{\rm hered} = \mathcal{F}_{\rm tail} + \mathcal{F}_{\rm tail(tail)} + \mathcal{F}_{\rm tail^2}\,, \tag{10}$$

with the quadratic order tails given as

$$\begin{aligned}\mathcal{F}_{\rm tail} &= \frac{4M}{5} I^{(3)}_{ij}(t)\int_0^{\infty} d\tau\, I^{(5)}_{ij}(t-\tau)\left[\ln\left(\frac{\tau}{2r_0}\right)+\frac{11}{12}\right] \\ &+ \frac{4M}{189} I^{(4)}_{ijk}(t)\int_0^{\infty} d\tau\, I^{(6)}_{ijk}(t-\tau)\left[\ln\left(\frac{\tau}{2r_0}\right)+\frac{97}{60}\right] \\ &+ \frac{64M}{45} J^{(3)}_{ij}(t)\int_0^{\infty} d\tau\, J^{(5)}_{ij}(t-\tau)\left[\ln\left(\frac{\tau}{2r_0}\right)+\frac{7}{6}\right]\,.\end{aligned} \tag{11}$$

and the cubic order tails as

$$\begin{aligned}\mathcal{F}_{\rm tail(tail)} &= \frac{4M^2}{5} I^{(3)}_{ij}(t)\int_0^{\infty} d\tau\, I^{(6)}_{ij}(t-\tau) \\ &\times \left[\ln^2\left(\frac{\tau}{2r_0}\right)+\frac{57}{70}\ln\left(\frac{\tau}{2r_0}\right)+\frac{124627}{44100}\right]\,,\end{aligned} \tag{12a}$$

$$\mathcal{F}_{\rm tail^2} = \frac{4M^2}{5}\left(\int_0^{\infty} d\tau\, I^{(5)}_{ij}(t-\tau)\left[\ln\left(\frac{\tau}{2r_0}\right)+\frac{11}{12}\right]\right)^2\,. \tag{12b}$$

$$h = h_{monopole} + h_{dipole} + h_{quadrupole} + \cdots.$$

h_{tail}

$$\frac{2GM}{c^3}\int_0^{+\infty} \mathrm{d}\tau\left[\ln\left(\frac{c\tau}{2r_0}\right)+\frac{11}{12}\right]\mathrm{M}^{(4)}_{ij}(U-\tau)$$

$h_{tail(tail)}$

$$2\left(\frac{GM}{c^3}\right)^2\int_0^{+\infty} \mathrm{d}\tau\left[\ln^2\left(\frac{c\tau}{2r_0}\right)+\frac{57}{70}\ln\left(\frac{c\tau}{2r_0}\right)+\frac{124627}{44100}\right]\mathrm{M}^{(5)}_{ij}(U-\tau)$$

$$\mathcal{F} \sim \left(h^{(2)}_{quadrupole} + h^{(2)}_{tail} + h^{(2)}_{tail(tail)} \cdots\right)^2$$

5. Summary

1. For Gravitational wave Astronomy, we need more accurate GW templates.

2. Hyperbolic encounter of compact binaries and gravitational waveform emitted from it are computed up to 3.5 PN accuracy.

3. 3PN accurate energy and angular momentum fluxes are computed.