Numerical Integration of Neutron Star Binaries in Eccentric Orbit

Kwan Ho Park

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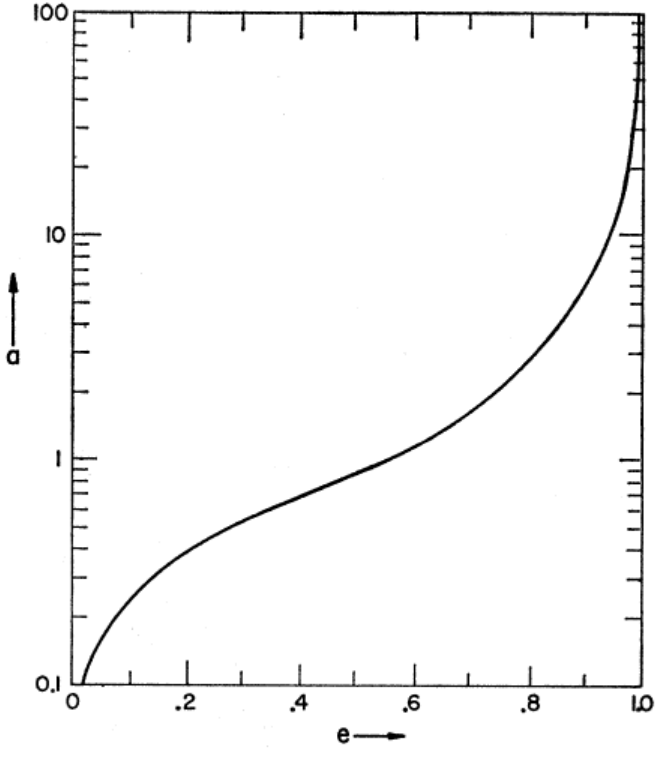
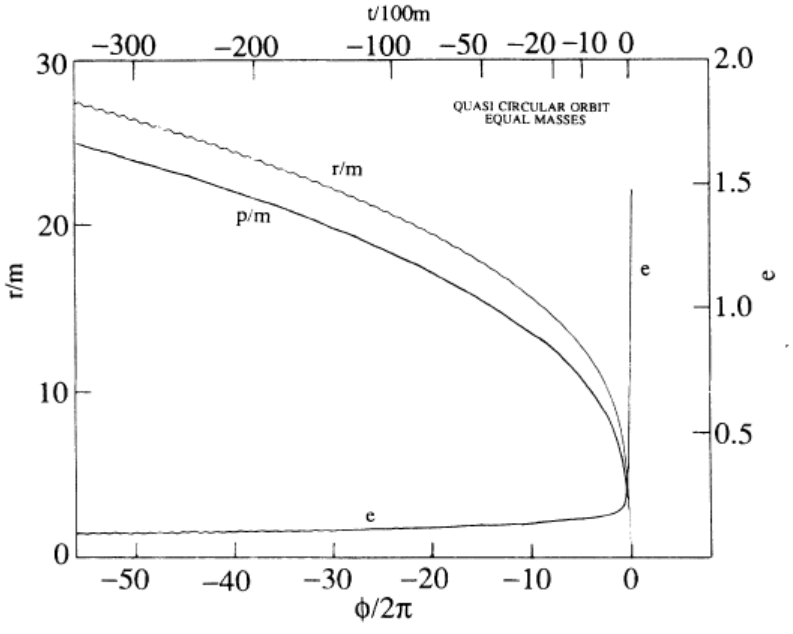
Physics & Astronomy

MOTIVATION



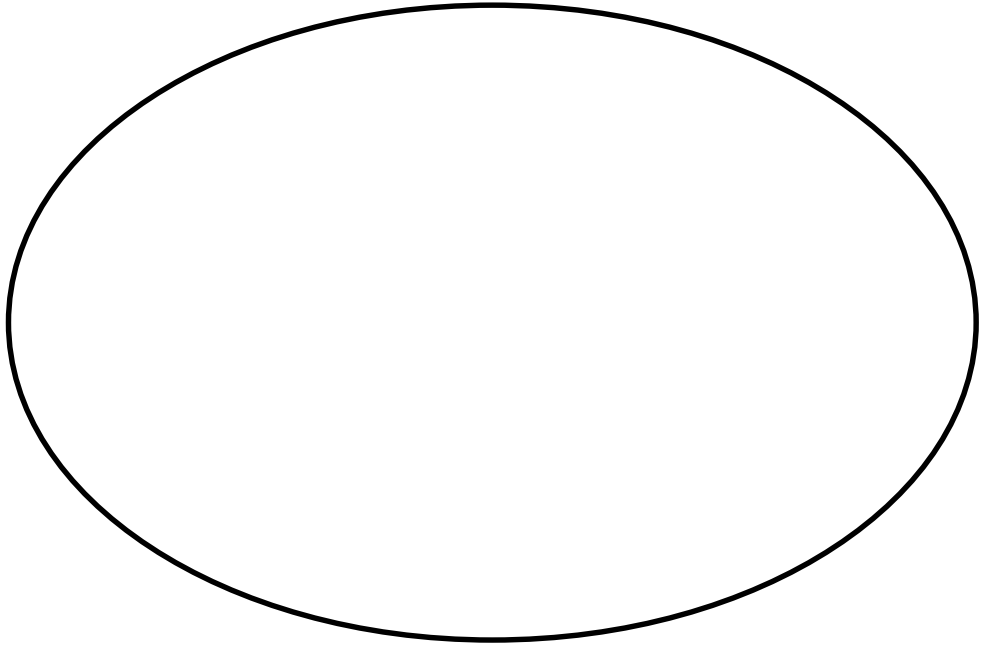
Peters, 1964



Lincoln & Will, 1990

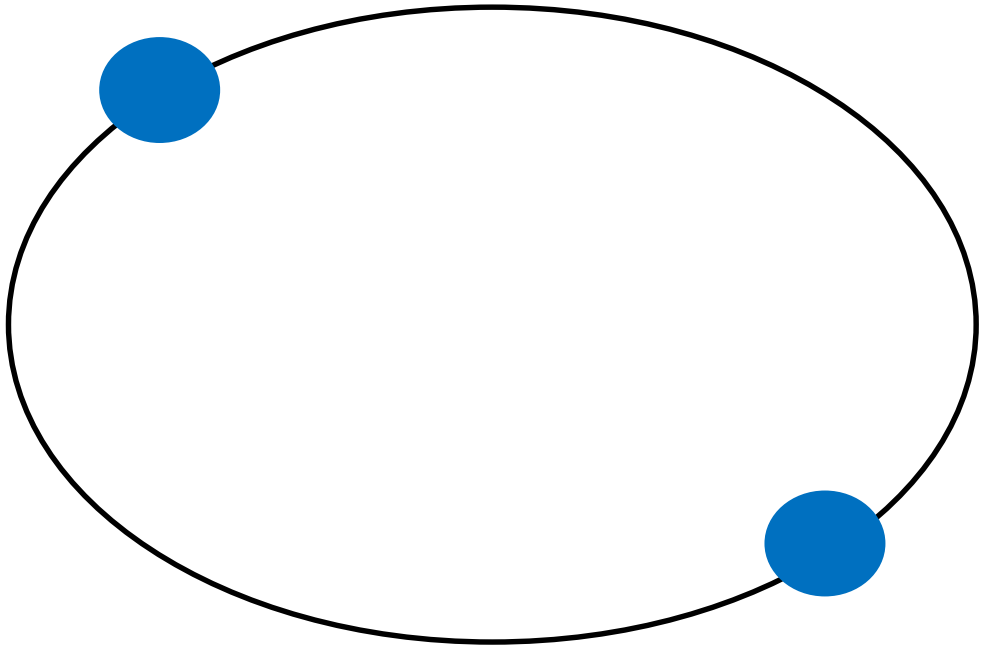
- Both of authors employed point particle and PN approximation.

BINARY NEUTRON STAR ANATOMY



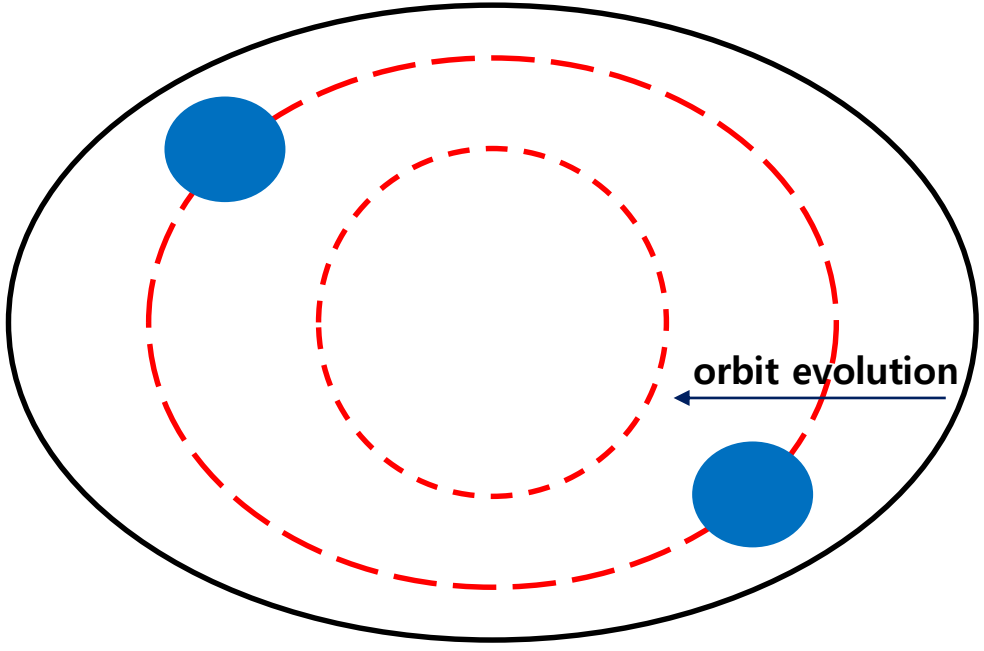
- Arbitrary eccentricity

BINARY NEUTRON STAR ANATOMY



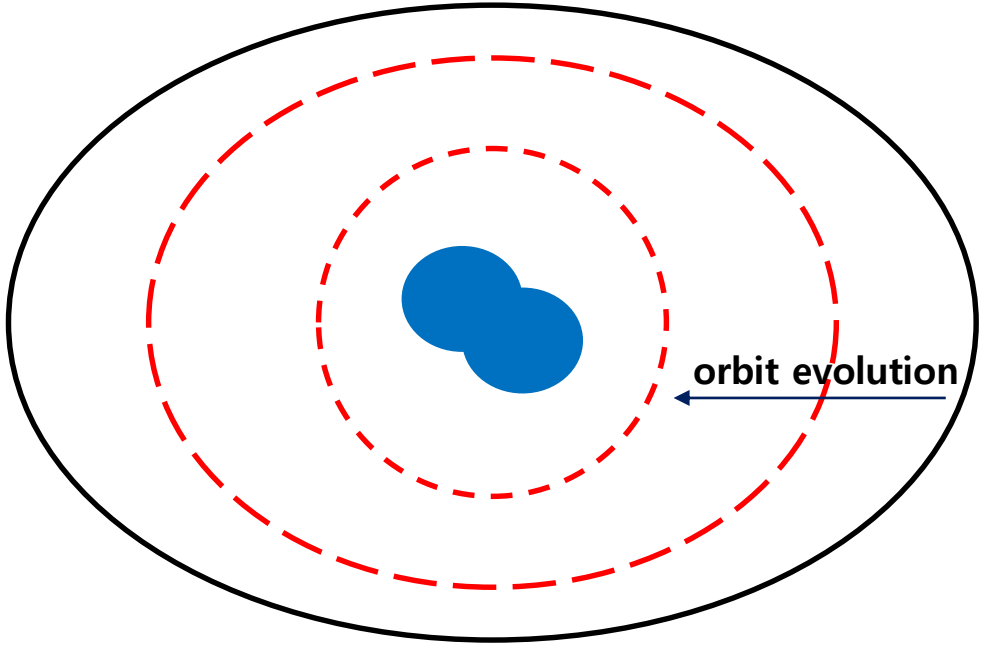
- Arbitrary eccentricity
- Deformed NS

BINARY NEUTRON STAR ANATOMY



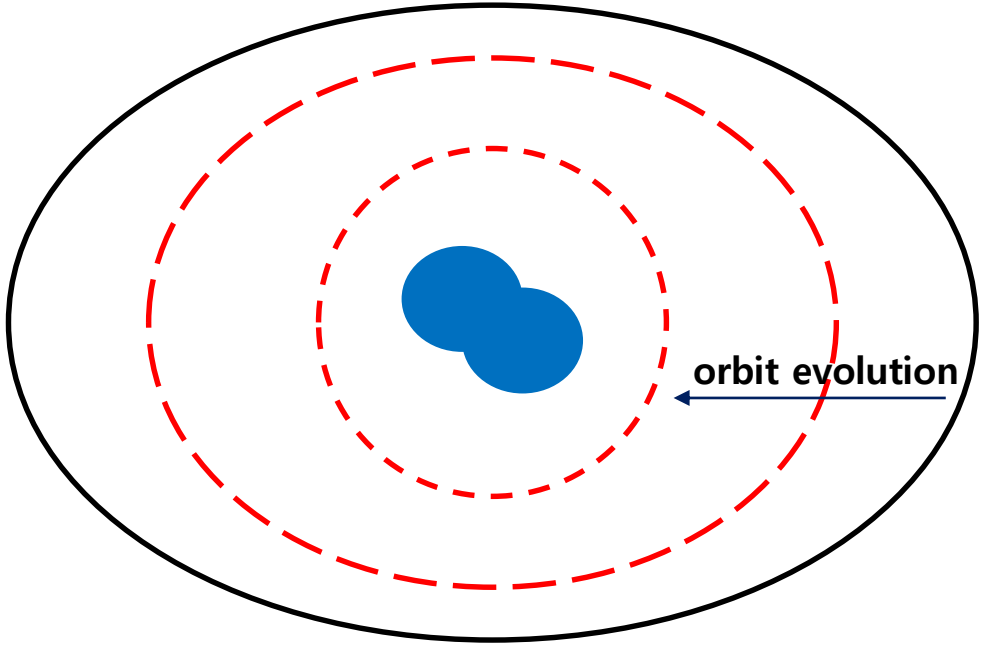
- Arbitrary eccentricity
- Deformed NS
- Gravitational wave
- Separation and eccentricity shrink.

BINARY NEUTRON STAR ANATOMY



- Arbitrary eccentricity
- Deformed NS
- Gravitational wave
- Separation and eccentricity shrink.
- Remnants

BINARY NEUTRON STAR ANATOMY



NR Initial data

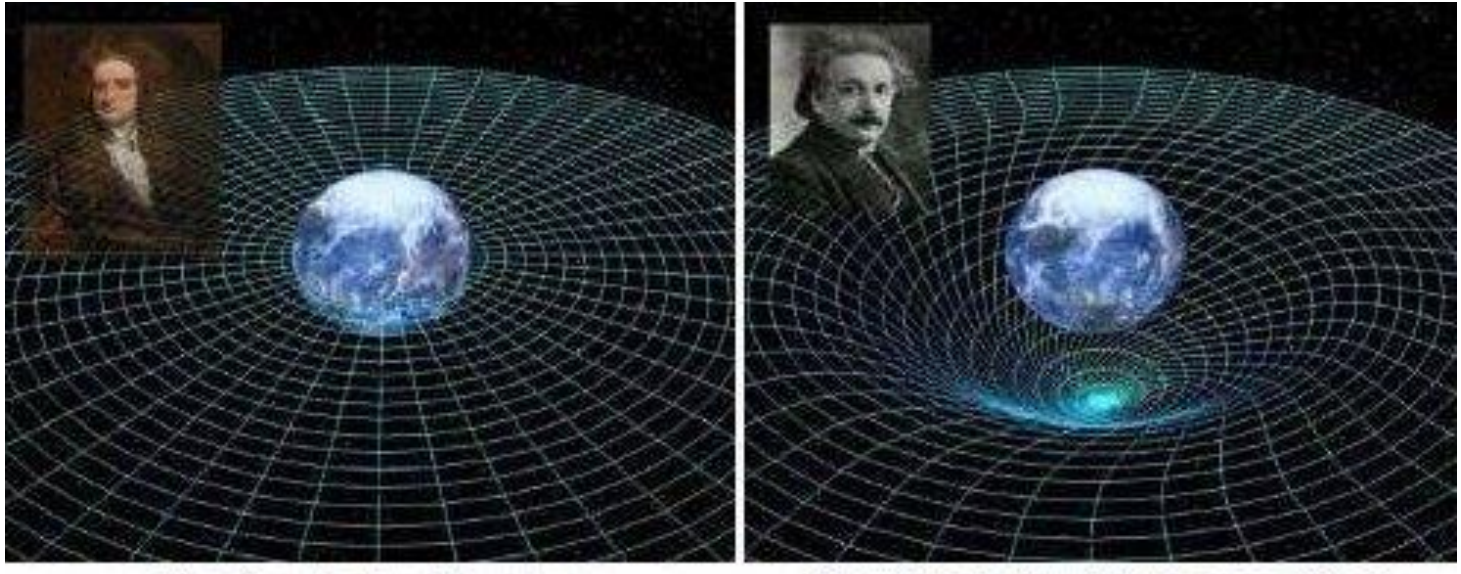
- Arbitrary eccentricity
- Deformed NS

- Gravitational wave
- Separation and eccentricity shrink.
- Remnants

NR Evolution

NR : Numerical Relativity

NS BINARY INITIAL DATA



https://www.nasa.gov

Einstein gravity needs ‘proper’ curvature matching to matter distribution.

NS BINARY INITIAL DATA

$$R + K^2 - K_{ij}K^{ij} = 16\pi H$$

$$D_j K_i^j - D_i K = 8\pi S_i$$

- H, S_i are given by matter distribution.
- K^{ij}, K, R are related to curvature of space-time.
- D_i represents differential operator.

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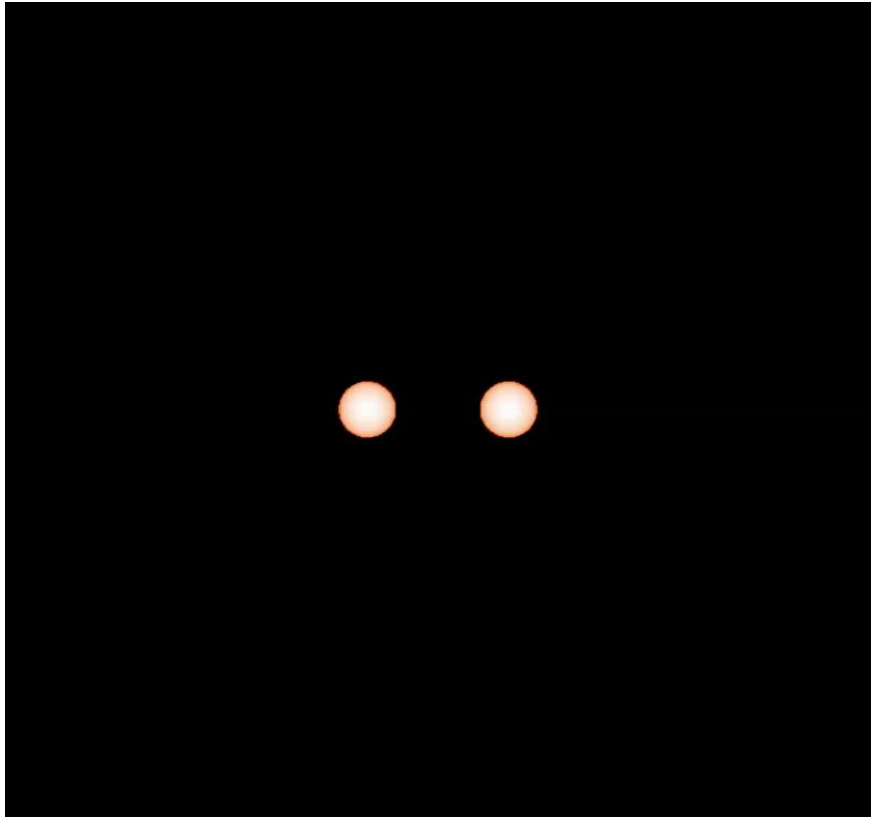
There is exact solution for quasi circular orbit.

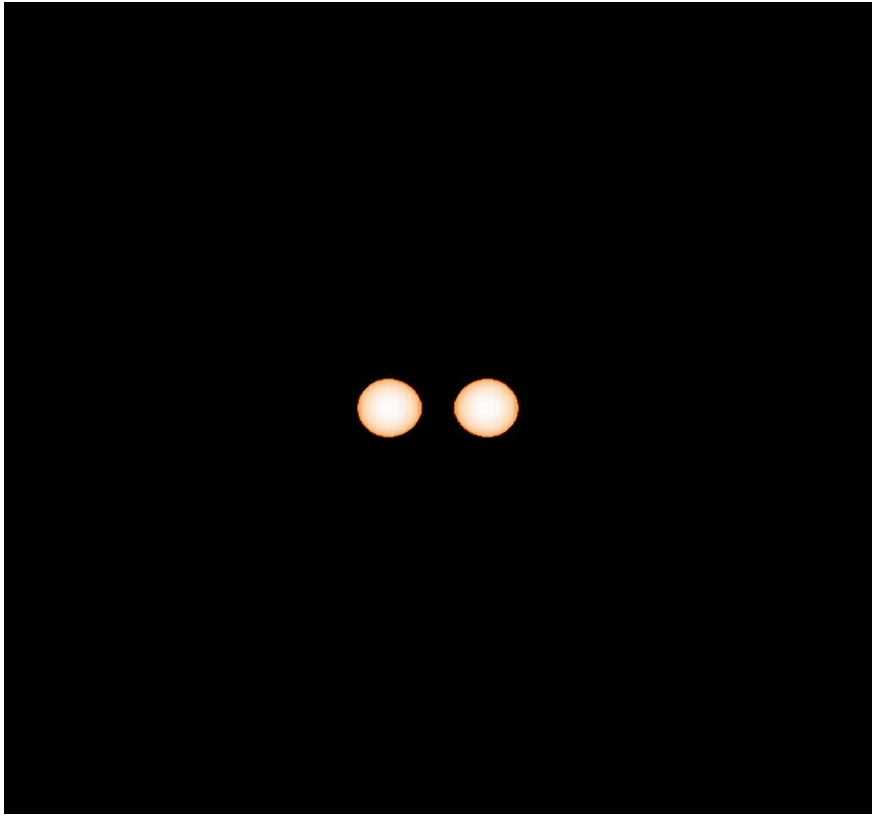
NS BINARY INITIAL DATA & EVOLUTION, QC



1.779 M_s, 60km, $P = K\rho^{\Gamma}$, $\Gamma = 2$

- Provided by LORENE
- Unrealistic EoS
- Prompt collapse

NS BINARY INITIAL DATA & EVOLUTION, QC



1.33 M_s, 45km, $P = K\rho^{\Gamma}$, $\Gamma = 2$

- Provided by LORENE
- Unrealistic EoS
- HMNS, delayed collapse

In QC, remnant depends on mass and EoS.

NS BINARY IN ECCENTRIC ORBIT

When we construct initial data for NS binary, we assume that

I. The orbit is stationary.

II. Fluid particle is irrotational flow.

III. Hydro variables share same symmetry of space – time.

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NS BINARY IN ECCENTRIC ORBIT

Circular orbit

Eccentric orbit

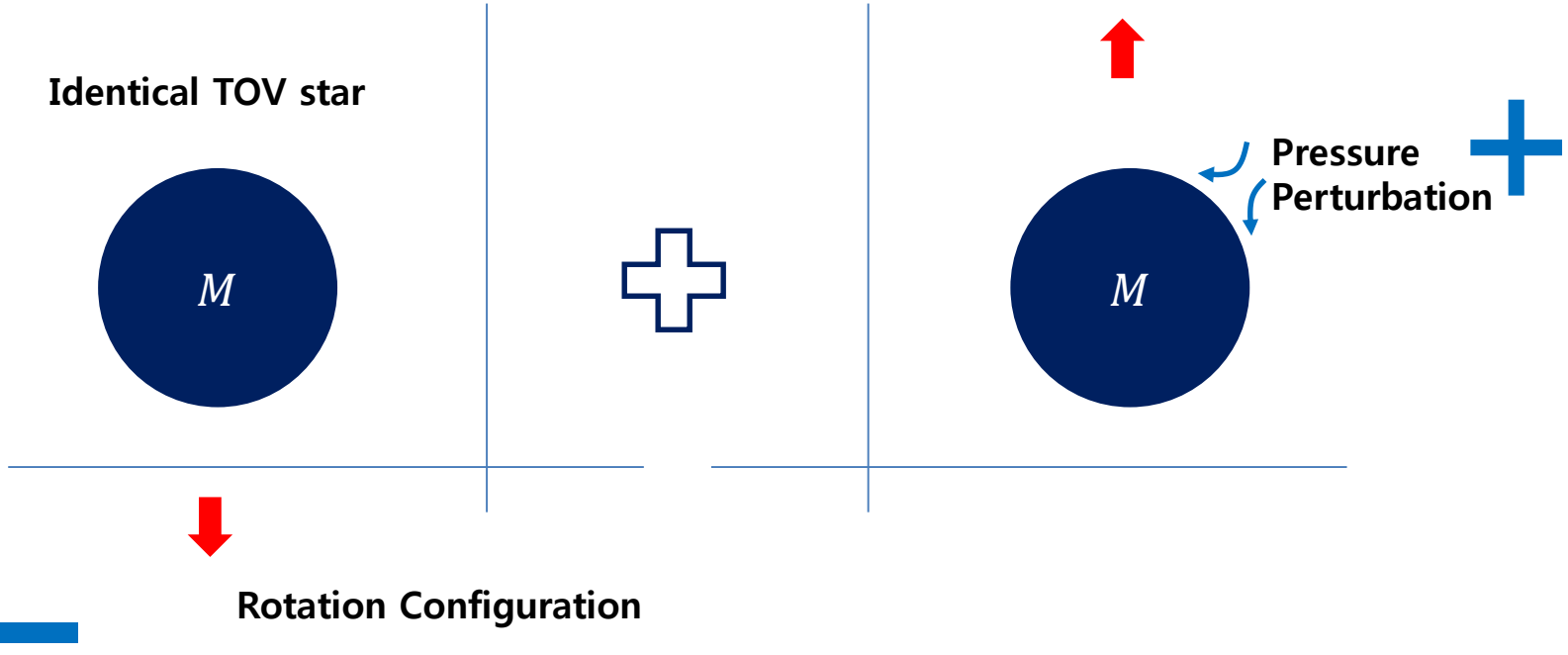
At rotating frame, NS binary keeps its shape in circular orbit. But NS binary in eccentric orbit can’t keep stellar surface due to separation changing.

NS BINARY IN ECCENTRIC ORBIT

I. Setting initial data **in a simple way** without solving constraints equations

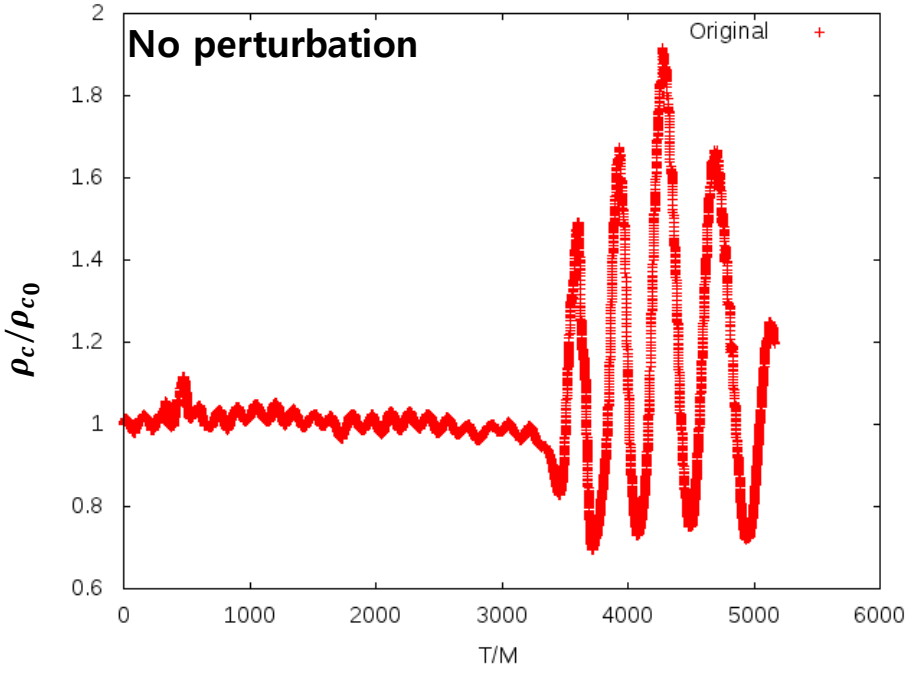
II. Evolution of eccentric orbits without unphysical behavior of binary system

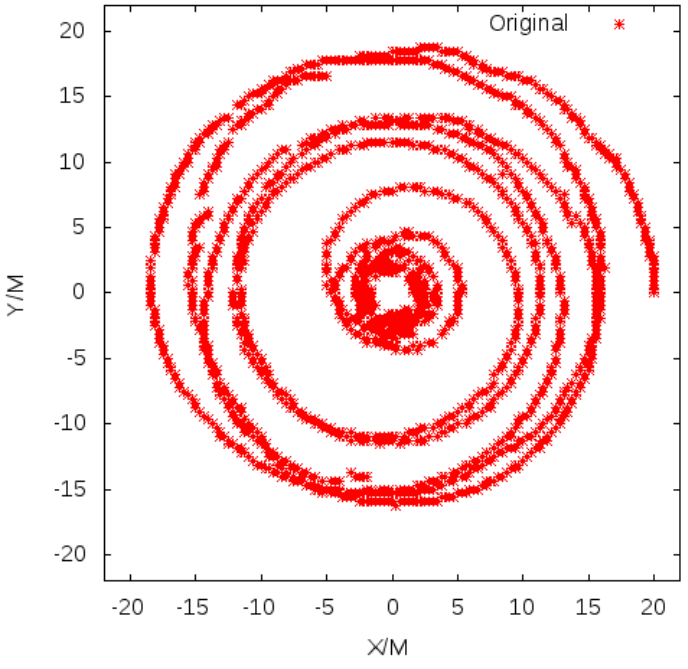
NS BINARY IN ECCENTRIC ORBIT



Such a configuration does not satisfy constraint equations.

NS BINARY IN ECCENTRIC ORBIT

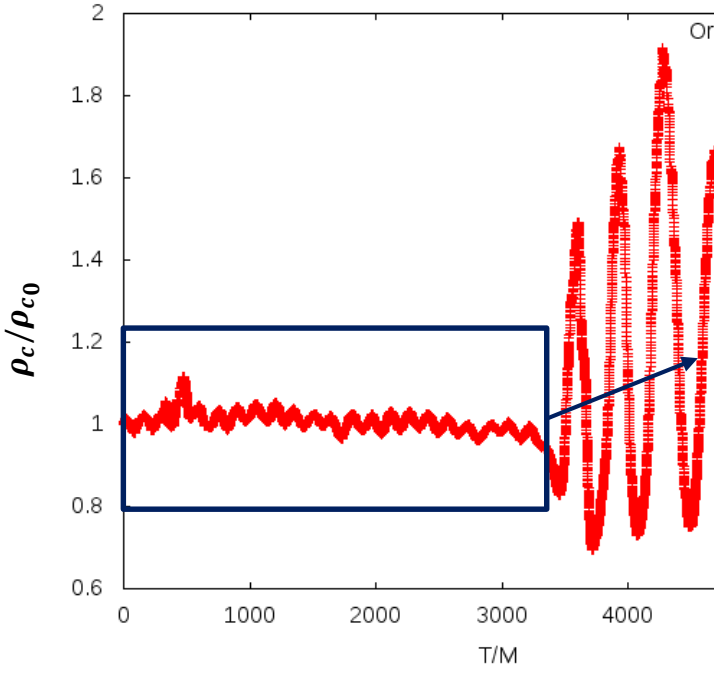


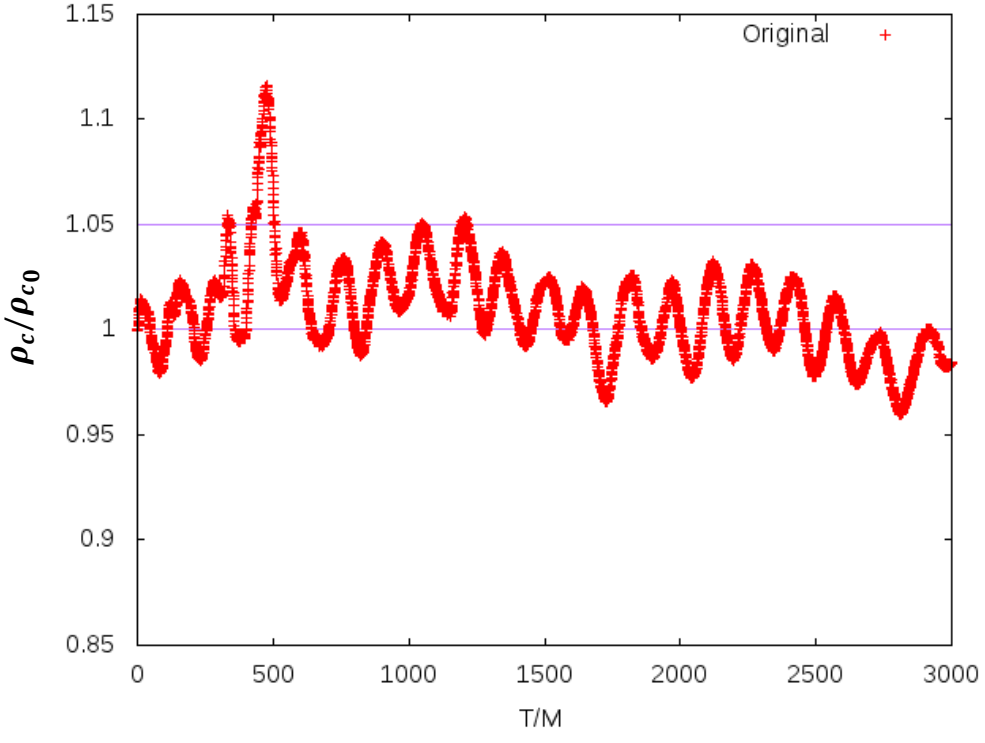


- Identical 1.4 M_s
- 58.8 km separation
- $P = K\rho^{\Gamma},\ \Gamma = 2 \quad e_N \sim 0.17$

- Trace of maximum density

NS BINARY IN ECCENTRIC ORBIT





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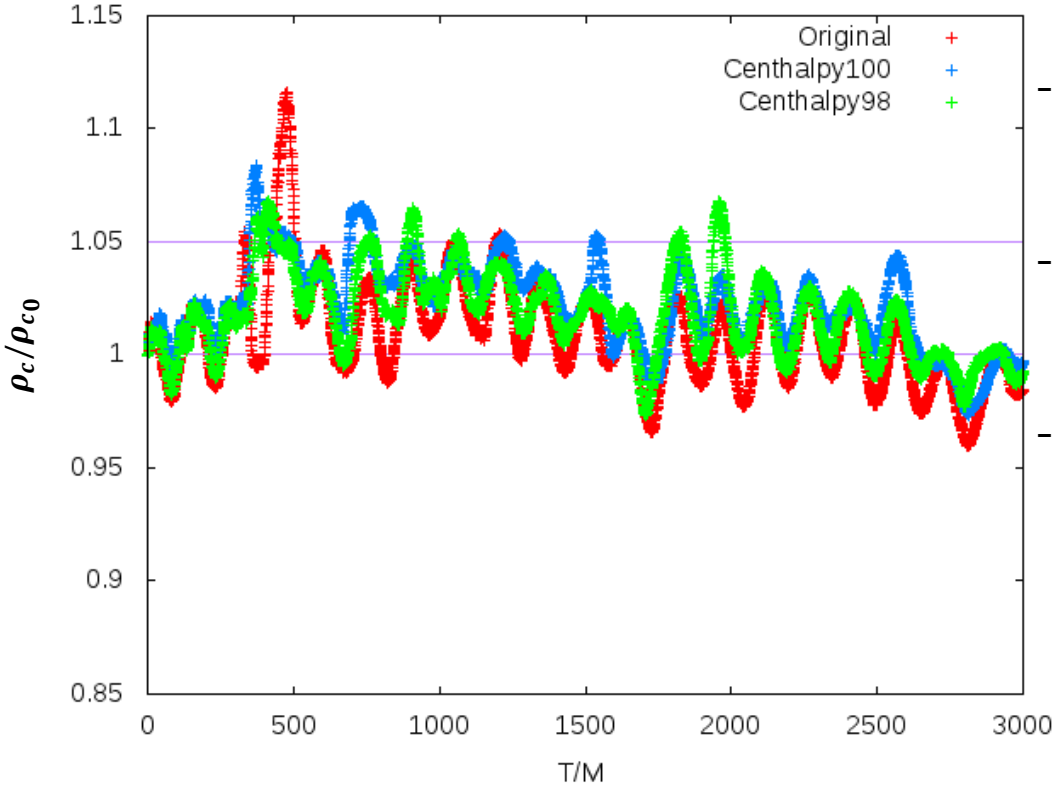
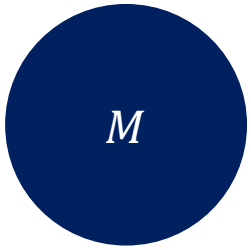


- High amplitude oscillation was significantly suppressed.
- Small amplitude oscillation still remains.
- Overall oscillation amplitude can be kept within 5% which is similar to quasi circular case.

THANKS!

APPENDIX

INITIAL DATA FOR ECCENTRIC ORBIT ∋ TOV star



Perfect fluid

$$T^{\mu\nu} = (e + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$$

Polytropic eqation of state

$$P = K\rho^{\Gamma}$$

Spherical symmetric metric field

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -e^{2\psi}dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

INITIAL DATA FOR ECCENTRIC ORBIT ∋ rotation

Vanishing vorticity asserts

$$\Omega_{\mu\nu} = \nabla_\nu(hu_\mu) - \nabla_\mu(hu_\nu) = 0$$

$$hu_\mu = \nabla_\mu \phi$$

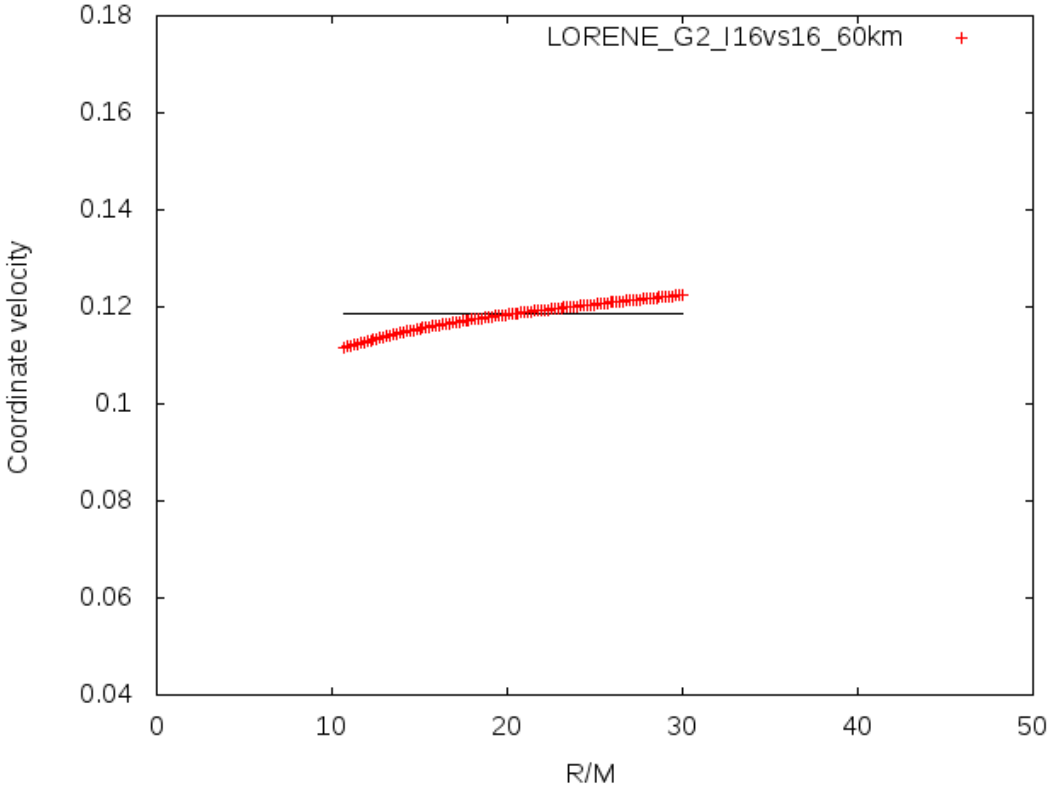
Solution comes from

$$\nabla_\mu \left(\frac{\rho}{h}\nabla^\mu \phi\right) = 0$$

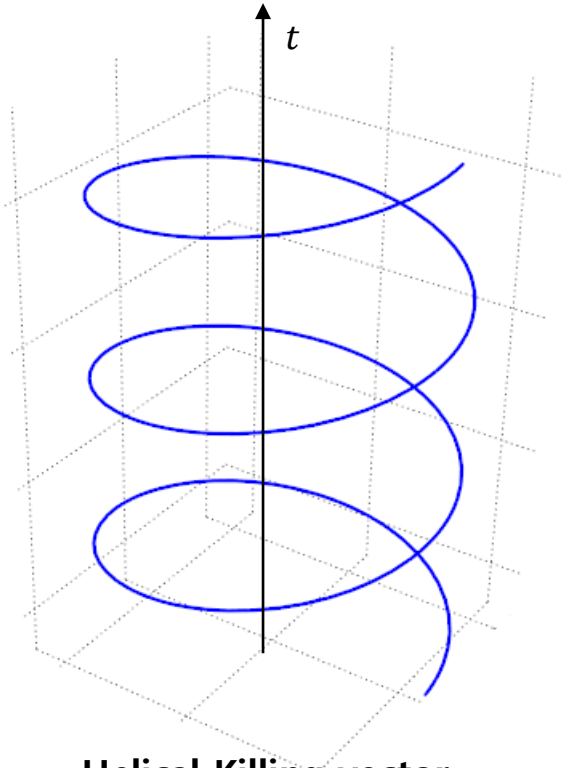
Irrotational fluid four velocity is approximately

$$\frac{u^i}{u^t} = C$$

Moldenhauer et al, 2014



INITIAL DATA FOR ECCENTRIC ORBIT ∋ perturbation



Helical Killing vector

When symmetry is given, injection energy is defined as

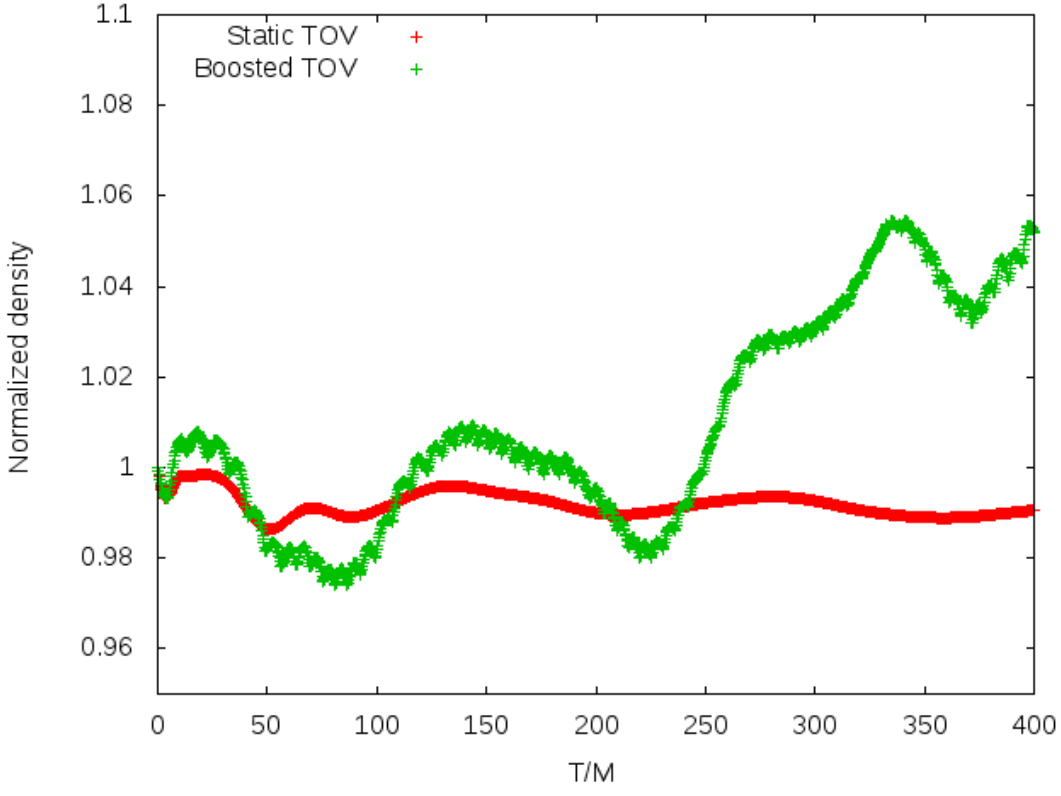
$$E \equiv h u_\mu \xi^\mu$$

Irrotational perfect fluid with helical symmetry ensures that

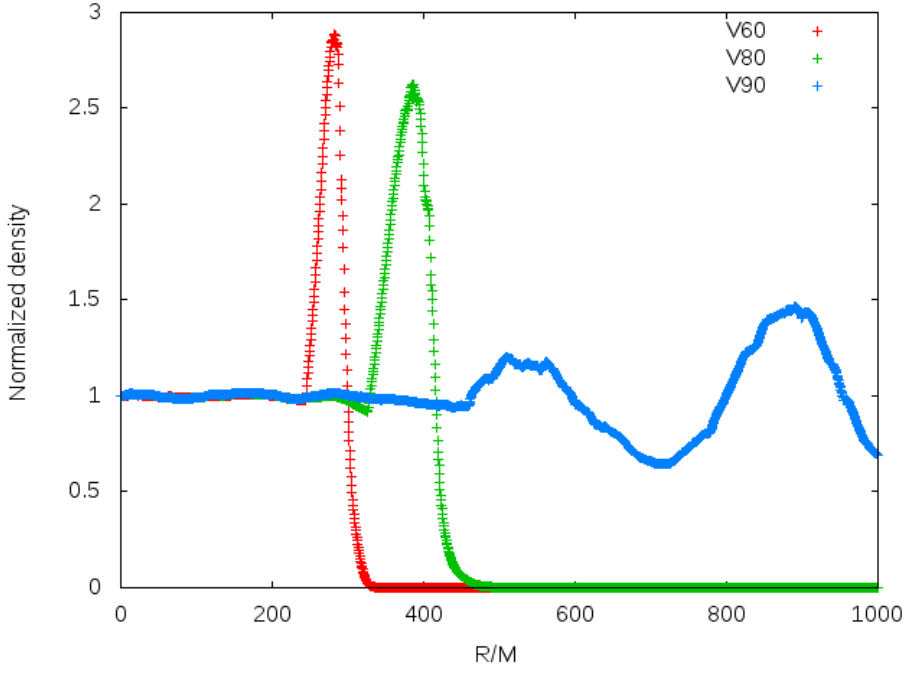
$$\nabla_\mu E = 0 \;\rightarrow\; u^\mu \nabla_\mu E = 0$$
$$\rightarrow\; \rho h u^\mu \nabla_\mu u_\nu + \left(\delta^\mu_\nu + u^\mu u_\nu\right)\nabla_\mu p = 0$$

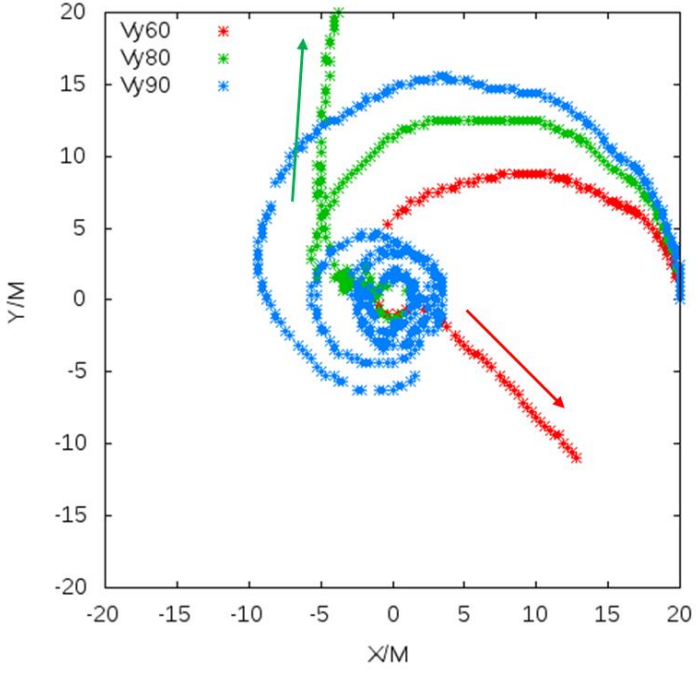
"Homogeneous injection energy promise relativistic force balance."

INITIAL DATA FOR ECCENTRIC ORBIT ∋ boosted TOV



ECCENTRIC ORBIT EVOLUTION ∋ high eccentricity orbit





ECCENTRIC ORBIT EVOLUTION ∋ quasi circular orbit