Post-Newtonian Energy Conservation in Algorithmic Chain Regularization

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Post-Newtonian Treatment

Decision-Making

Case Studies

Hierarchical Systems

Post-Newtonian Terms

Equation of motion $$\frac{d^2\mathbf{r}}{dt^2} = \frac{M}{r^2}\left[(-1+A)\frac{\mathbf{r}}{r} + B\mathbf{v}\right]$$

GR formulation Blanchet & Iyer 2003

First-order precession $$M = m_1 + m_2, \quad \eta = \frac{m_1 m_2}{M^2}$$

$$A_1 = 2(2+\eta)\frac{M}{r} - (1+3\eta)v^2 + \frac{3}{2}\eta\dot{r}^2$$

$$B_1 = 2(2-\eta)\dot{r}$$

Higher-order precession $$A_2 = ..., \quad B_2 = ..., \quad A_3 = ..., \quad B_3 = ...$$

Gravitational radiation $$A_{5/2} = \frac{8}{5}\eta\frac{M}{r}\dot{r}\left(\frac{17M}{3r} + 3v^2\right)$$

$$B_{5/2} = -\frac{8}{5}\eta\frac{M}{r}\left(3\frac{M}{r} + v^2\right)$$

Total GR perturbation

$$\mathbf{P}_{GR} = \frac{M}{c^2 r^2}\left[(A_1 + \frac{A_2}{c^2} + \frac{A_{5/2}}{c^3})\frac{\mathbf{r}}{r} + (B_1 + \frac{B_2}{c^2} + \frac{B_{5/2}}{c^3})\mathbf{v}\right]$$

Energy check $$E_{\rm tot} = \mathrm{E_{kin}} + \mathrm{E_{pot}} + \mathrm{E_{bin}} + \mathrm{E_{ch}} - \int \mathbf{P}_{\rm GR}\cdot\mathbf{v}dt = \text{const}$$

GR radiation time-scale $$t_{\rm GR} = \frac{5}{64}\frac{c^5\, g(e)\, a^4}{X(1+X)\, m_1^3}, \quad c = \frac{3\times 10^5}{V^*}$$

$$g(e) \simeq \frac{(1-e^2)^{7/2}}{4.35}, \quad X = \frac{m_2}{m_1}$$

Decision-Making

Increasing GR effect $\quad t_{\rm GR} \;<\; 500,\;\; 50,\;\; 1$

$\qquad\qquad\qquad\qquad\quad \mathrm{IPN} \;=\; 1,\;\; 2,\;\; 3$

Einstein shift $\quad \Delta w = \dfrac{6\pi M}{ac^2(1-e^2)}$

Graduated scale $\quad \Delta w > (1, 10, 100) \times 10^{-4} \;\; \Rightarrow \mathrm{IPN} = 1,\;\; 2,\;\; 3$

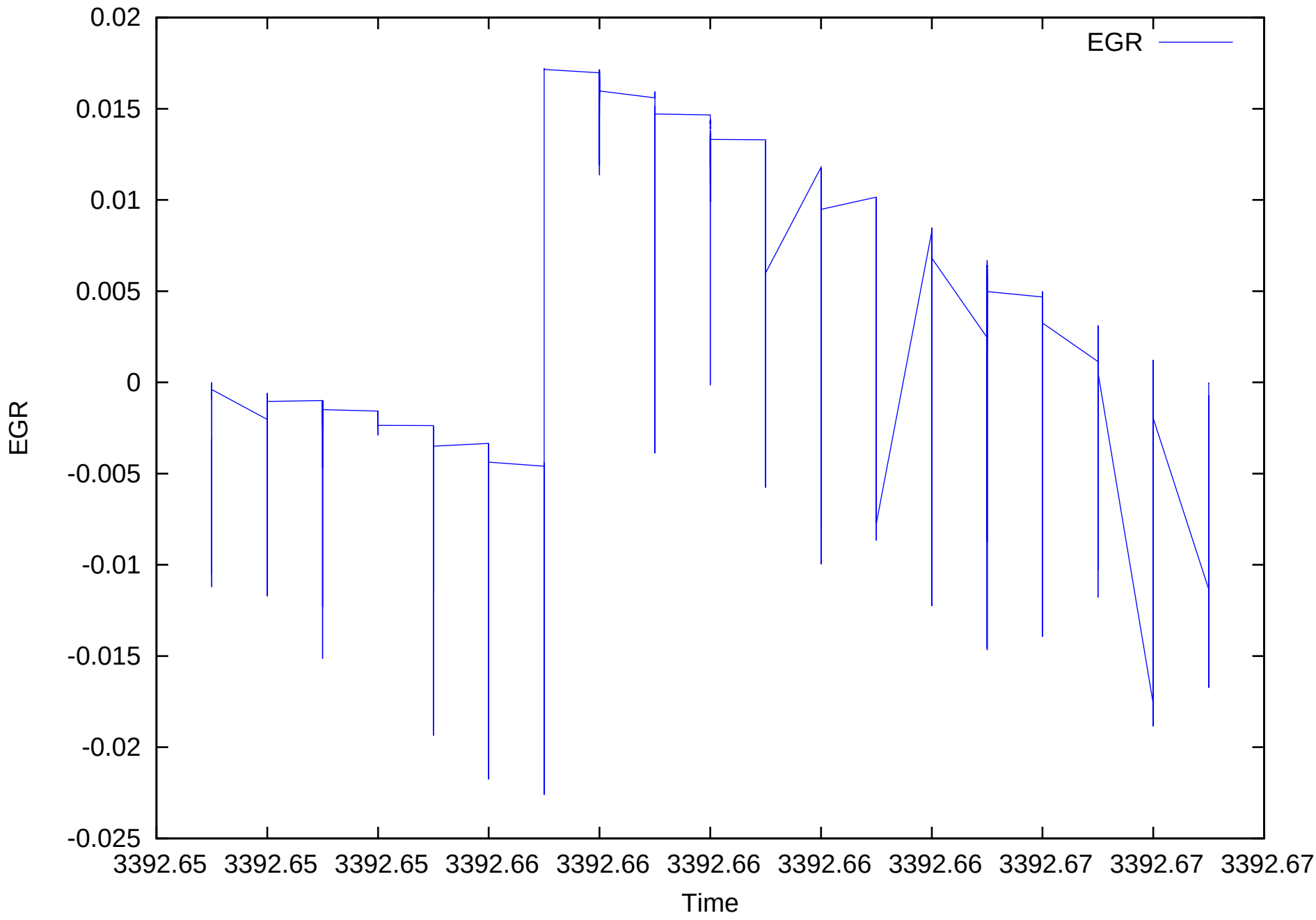
Coalescence $\quad R \;<\; 4R_{\rm Sch} \;=\; \dfrac{8\,M}{c^2}$

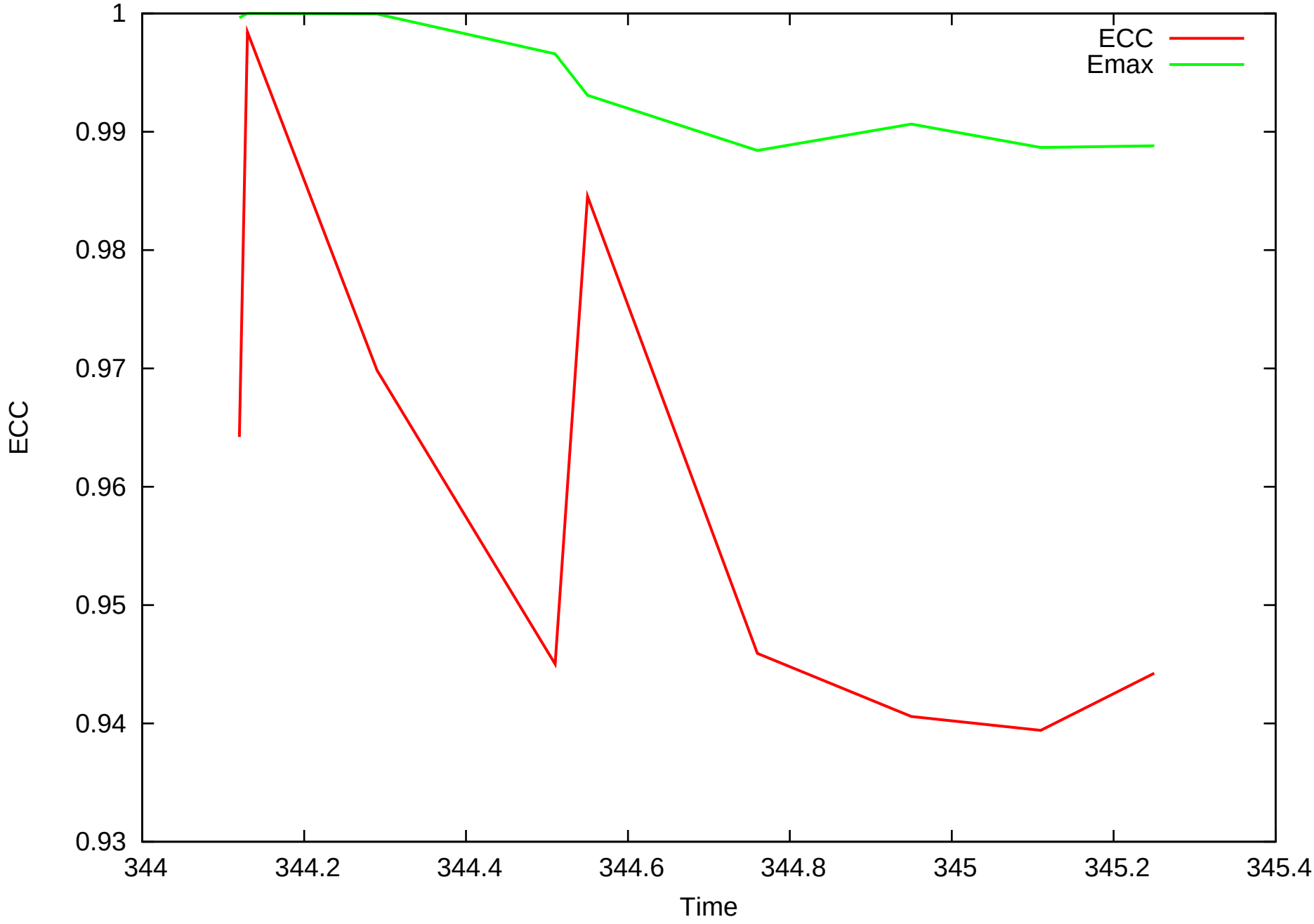
Alternative merging $\quad \mathrm{IPN} = 3, \;\; N = 2, \;\; N_{\rm p} = 0$

$\qquad\qquad\qquad\qquad\quad \mathrm{IPN} \geq 2, \;\; a\,(1-e) < R_{\rm Sch}$

$\qquad\qquad\qquad\qquad\quad \mathrm{IPN} = 3, \;\; N = 3, \;\; a_1\,(1-e_1) > 100\,a$

Unperturbed KS $\quad t_{\rm GR} < 500 \;\&\; \Delta w > 1{\times}10^{-4} : \;\; \dot{a},\; \dot{e} \;\&\; \Delta w \;\; \Rightarrow \mathbf{u},\; \mathbf{u}'$


EGR
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
-0.025
EGR
3392.65
3392.65
3392.65
3392.66
3392.66
3392.66
3392.66
3392.66
3392.67
3392.67
3392.67
Time


ECC
Emax
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
ECC
344
344.2
344.4
344.6
344.8
345
345.2
345.4
Time

Case Studies

Basic parameters $N = 50,000$, IMF $0.1 - 50\ M_{\odot}$, $N_{bh} = 20$

Velocity of light $c = 1000,\ 8000,\ 20,000,\ 30,000$

Model A $c = 1000,\ e = 0.89$ (*)

Model B $c = 8000,\ e = 0.988$ (*)

Model C $c = 20,000,\ e = 0.9999$ (*)

Model D $c = 20,000,\ e = 0.9992$ (*)

Model E $c = 20,000,\ e = 0.9997,\ e_x = 0.99999$ (*)

Model F $c = 30,000,\ e = 0.99998,\ e_x = 0.99998$

(*) Coalescence

Long-lived Hierarchy

ECC	E_{max}	Inc
0.867	0.9996	91.3
0.978	0.9994	86.0
0.928	0.9998	89.1
0.921	0.9999	89.2
0.879	0.9994	91.5
0.911	0.9997	91.2
0.890	0.9998	91.0
0.921	0.9998	89.0

Configuration	Three black holes, $M = 24, 24, 26$
Einstein shift	$\Delta w = 3 \times 10^{-3}$, IPN $= 2$, $c = 30,000$
Time scale	10^4 block $-$ steps, $\Delta T = 0.52$